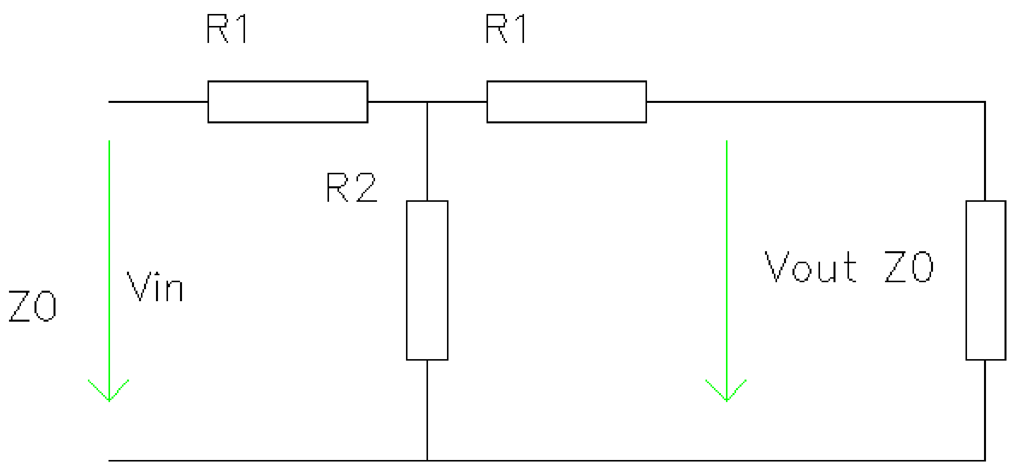
Deriving T-attenuator formulae



$$V_{OUT} = \frac{Z_0}{R_1 + Z_0} \frac{R_2 \parallel (R_1 + Z_0)}{R_1 + R_2 \parallel (R_1 + Z_0)} V_{IN} \tag{1}$$

$$A = \frac{V_{OUT}}{V_{IN}} = Z_0 \frac{R_2 \parallel (R_1 + Z_0)}{(R_1 + Z_0)(R_1 + R_2 \parallel (R_1 + Z_0))} \tag{2}$$

Because Input impedance of the attenuator must be equal to Z_0

$$R_1 + R_2 \parallel (R_1 + Z_0) = Z_0 \tag{3}$$

Substitute $R_2 \parallel (R_1 + Z_0) = Z_0 - R_1$ in (2)

$$A = Z_0 \frac{Z_0 - R_1}{(R_1 + Z_0)(\cancel{R_1} + Z_0 \cancel{- R_1})} = \frac{\cancel{Z_0}(Z_0 - R_1)}{\cancel{Z_0}(R_1 + Z_0)} \tag{4}$$

Solve R_1

$$AR_1 + AZ_0 = Z_0 - R_1 \rightarrow \boldsymbol{R_1 = Z_0 \frac{(1 - A)}{(1 + A)}} \tag{5}$$

Write out the parallel impedance and substitute R_1 in (3)

$$\frac{R_2(R_1+Z_0)}{R_2+(R_1+Z_0)} = Z_0 - R_1 \rightarrow \frac{R_2\left(\left(Z_0\frac{(1-A)}{(1+A)}\right)+Z_0\right)}{R_2+\left(\left(Z_0\frac{(1-A)}{(1+A)}\right)+Z_0\right)} = Z_0 - \left(Z_0\frac{(1-A)}{(1+A)}\right) \rightarrow$$

$$\frac{\frac{R_2Z_0 - R_2Z_0A}{(1+A)} + R_2Z_0}{R_2 + \frac{Z_0(1+A)}{(1+A)} + Z_0} = Z_0\left(1-\frac{(1-A)}{(1+A)}\right) \rightarrow$$

$$\frac{\frac{R_2Z_0 \cancel{- R_2Z_0A} + R_2Z_0 + \cancel{R_2Z_0A}}{\cancel{(1+A)}}}{\frac{R_2+R_2A+Z_0(1-A)+Z_0+Z_0A}{\cancel{(1+A)}}} = Z_0\left(1-\frac{(1-A)}{(1+A)}\right) \rightarrow$$

$$\frac{2R_2\cancel{Z_0}}{R_2(1+A)+2Z_0} = \cancel{Z_0}\left(1-\frac{(1-A)}{(1+A)}\right) \rightarrow$$

$$2R_2 = [R_2\,(1+A)+2Z_0]\left(1-\frac{(1-A)}{(1+A)}\right)$$

$$= R_2\,(1+A) - R_2\,(1-A) + 2Z_0 - 2Z_0\frac{(1-A)}{(1+A)} \rightarrow$$

$$\cancel{2}R_2 = \cancel{2}R_2A + \cancel{2}Z_0\frac{(1-A)}{(1+A)} \rightarrow R_2 = R_2A + Z_0\left(1-\frac{(1-A)}{(1+A)}\right)$$

$$= R_2A + Z_0\left(\frac{\cancel{1}+A-\cancel{1}+A}{1+A}\right) = R_2A + Z_0\left(\frac{2A}{1+A}\right) \rightarrow$$

$$R_2(1-A) = 2Z_0\frac{A}{(1+A)(1-A)} \rightarrow \boldsymbol{R_2 = 2Z_0\frac{A}{(1-A^2)}} \qquad (6)$$